

Mr. Keynes and the "Classics" Again: An Alternative Interpretation

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It will be admitted by the least charitable reader that the textbook exposition of Keynesian economics and its integration with Classical economics are elegant. But it is also clear to most people who have read Keynes that the AS/AD textbook exposition of Keynesian economics embodies ideas that are not to be found in *The General Theory*, and includes others that Keynes went to great lengths to repudiate. In this paper I offer an alternative AS/AD exposition of Keynesian economics which better captures the essence of Keynes' ideas and offers a superior integration of Keynesian economics with Classical economics.

The underlying ideas expressed in my alternative exposition are not original. They can be found, in varying degrees, in the work of Keynes, Don Patinkin, Abba Lerner, James Tobin, Paul Davidson, Robert Clower, Axel Leijonhufvud, and some of the New Keynesians, to mention only a few of those who have provided richer alternative interpretations of Keynesian economics. But these alternative interpretations are known primarily by macroeconomic and history of thought specialists. They have not become part of the standard core of macroeconomic theory as presented in the introductory and intermediate texts. Indeed, as I can testify to by experience, any attempt to include a discussion of these alternative interpretations in introductory or intermediate texts is met with great hostility by reviewers as being outside of the standard core.

Despite the multitude of interpretations of Keynesian macroeconomics, only three have become part of the core of macroeconomics: Samuelson's Keynesian Cross, Hicks' IS/LM model, and a generic aggregate supply-demand model. Any interpretation which cannot be captured in one of these models is not part of the core macro.

Having followed the interpretative debates, I have become a strong believer in the power of a simple model and in the proposition that a simple model can only be replaced by an equally simple model. Any interpretation that cannot be embodied in a simple two-dimensional geometric model will be relegated to history of thought, not to the core of macroeconomic theory. Following this proposition, the initial acceptability of Keynesian ideas in the United States classrooms was in large part due to Alvin Hansen's and Paul Samuelson's ability to embody it in the Keynesian Cross multiplier model. Similarly, the stability of the Keynesian neo-Classical synthesis is in large part due to Hicks' neat IS/LM diagram and the aggregate supply/aggregate demand additions to that model.

The profession's enormous demand for simplicity has led to the AS/AD model becoming the core of introductory economics, and most economists' initial thinking about the macroeconomy. In fact, in most introductory textbooks AS/AD is replacing the Keynesian Cross as a technically superior way to present ideas about the aggregate economy.

The hypothesis of this paper is *that that standard AS/AD model of the aggregate economy embodies a fundamentally incorrect view of the nature of Keynesian economics*, and about the way the macroeconomy operates. That hypothesis is one that I believe is shared by many macroeconomic specialists. My contribution in this paper is to provide an equally simple, yet richer, AS/AD model of the macroeconomy which yields an interpretation of Keynesian economics which is more consistent with recent developments and which better focuses on the differences between the Keynesian and classical views of equilibrium of the aggregate economy. As will become clear below, explaining this alternative model exposes an inconsistency between

Mr. Keynes and the Classics Again

the AS/AD model, as currently interpreted, and the Keynesian Cross model. One or the other is correct, but not both. My alternative eliminates that inconsistency.

My alternative AS/AD model is close to the current model, but it embodies some fundamentally different interpretations and assumptions about the way the aggregate economy works. The most important of these are the reinterpretation of the aggregate demand curve, making the definition of the curve one which is independent of supply, and the addition of an explicit psychological or expectational element to supply or production which captures an aggregate interdependence between supply and demand decisions in the aggregate and which causes allows the supply curve to shift in response to expectations in demand. These two additions are what makes my analysis consistent with the Keynesian Cross and with many of the modern New Keynesian interpretations.

Specifically, my alternative model gives one a fundamentally different view of: (1) the nature of short-run equilibrium and the disequilibrium adjustment process that accompanies it; (2) the stability of the aggregate supply curve and the interrelationship between aggregate supply and demand; and (3) the role of wage flexibility in causing unemployment.

THE STANDARD AS/AD EXPOSITION

The standard AS/AD model is presented in Figure 1. The price level is on the vertical axis; total output is on the horizontal axis.

Figure 1

The Standard AS/AD Model

The Aggregate Demand Curve

The AD curve in the standard model is derived from the IS/LM model. To arrive at it one goes through a thought experiment, asking what the real income equilibrium would be at various price levels. The resulting set of equilibria are plotted in price quantity space and called an AD curve.

This procedure is not wrong, but it is problematic. To call this set of points an aggregate demand curve assumes that the IS/LM model only determines the level of aggregate demand--not the level of aggregate equilibrium income. But the IS/LM model is a model of equilibrium income, *not of aggregate demand*. In the IS/LM model effective demand determines the equilibrium output in the economy and is determined by an assumed interaction between aggregate supply and demand which works its way through a multiplier process. Calling the curve derived from this thought experiment an AD curve, not an aggregate equilibrium curve, is inconsistent with the standard definition of the IS curve as representing goods market equilibria.

The inconsistency could, of course, simply be a problem of labelling which could be solved by not calling it an AD curve but by calling it an Aggregate Equilibrium curve; it is a curve that captures the various equilibria of the economy at various price levels. Many Keynesians I have spoken with, when pressed on this issue, have said that they have always

Mr. Keynes and the Classics Again

interpreted it as an Aggregate Equilibrium curve.¹ Unfortunately, the problem is more severe than a simple labeling problem. This can be seen by asking the questions: If the AD curve is actually an Aggregate Equilibrium curve, what is the role of the AS curve? Isn't equilibrium determined by both aggregate supply and aggregate demand? *To add an AS curve to an Aggregate Equilibrium curve and to use both to determine an equilibrium is a contradiction.* Since an AS curve can't be combined with an aggregate equilibrium interpretation of the AD curve, let us now turn to the AS curve in the standard presentation.

The AS Curve

The standard AS curve is derived from a standard aggregate production function, which is seen as a simple sum of individual firms' production functions. Since in the standard model labor is the only variable input, the assumption about nominal wages is the primary determinant of the slope of the AS curve.

Two cases are normally distinguished. The supply curve in the short-run Keynesian case is upward sloping due to fixed nominal wages which cause labor market disequilibrium; in the Classical case and in the Keynesian long run, nominal wages are flexible and the labor market is in equilibrium, so the AS curve is perfectly inelastic. Long-run equilibrium is at the output level that corresponds to the natural rate of output, and at the price level determined by the quantity of money in the economy. Short-run equilibrium differs from long-run equilibrium only because of fixed nominal wages; equilibrium output can only exceed the natural rate of output if workers experience money illusion.

There is no logical problem with this interpretation: The problem comes when this aggregate supply curve is combined with the standard AD curve and the two are used to discuss disequilibrium adjustment. As I argued above, doing so is inconsistent with the aggregate equilibrium interpretation of the AD curve.

One way out of this dilemma is to take the position that the AS curve was never meant to be combined with the AD curve, or used to discuss disequilibrium adjustment. Instead, the AS curve was meant to be interpreted as a type of aggregate output constraint which tells us the maximum equilibrium output that is consistent with the supply conditions in the economy.² In this interpretation the AS curve is not the quantity that would be supplied at various prices; it is the maximum level of equilibrium output that could be achieved under specified conditions. Technically, such an interpretation is possible in the sense that the standard AS/AD equilibrium is the equilibrium arrived at by the standard set of equations that ultimately underlie the curves. Thus, I agree that one can resolve the logical inconsistency of the current AS/AD analysis if one doesn't interpret the AS curve as an aggregate supply curve and doesn't interpret the AD curve as an aggregate demand curve, and one considers only the equilibrium position determined by the intersection of the two curves. But to have an AS curve which isn't really a supply curve, and an AD curve which isn't really a demand curve leaves one wondering what role they play in the analysis.

Nonetheless, such a Humpty Dumpty situation is acceptable to many theoretical Keynesians because their world doesn't revolve around the teaching of principles of macro where the AS/AD analysis is the students' entry into macroeconomic thinking. They simply accept the model with little thought about its interpretation. In working out the implications of macro models, most theoretical Keynesians think in terms of solutions to simultaneous equations, not in terms of common sense, intuition, and stories of how the aggregate economy is likely to adjust to

¹ I would like to thank Robert Solow for patiently explaining this interpretation to me.

² I would like to thank James Tobin for explaining this interpretation of the AS curve to me.

Mr. Keynes and the Classics Again

shocks. Students and most teachers of economics don't think in terms of solutions to simultaneous equations; they think in terms of stories. Thus, in the principles world, all these subtle niceties of interpretation disappear and the AS and AD curves are used as the aggregate counterparts of partial equilibrium supply and demand curves. Indeed, it is precisely because AS/AD analysis makes students feel comfortable because of its similarity to partial equilibrium analysis that it developed, over the strong objections of a number of excellent textbook writers who ultimately succumbed to save market share. The market demanded AS/AD analysis and textbook authors had to respond.

Unfortunately textbook authors do not go through the subtleties of interpretation of the curves. At most, they make a qualifying addendum. Moreover, textbook authors had to address the questions that students were interested in--disequilibrium adjustment--and they had to provide answers using those curves. Even though the standard curves can't be used to discuss disequilibrium adjustment textbook authors were forced to use the curves incorrectly, and in every principles text I have examined, including Samuelson and Nordhaus, McConnell, Miller, Hyman, and in most intermediate macro texts, the AS/AD analysis is used incorrectly to discuss disequilibrium adjustment, even though it is inappropriate to use to discuss such issues.

The best way to see what's wrong with the present model is to use it to analyze a simple shift in aggregate demand of a specified amount, say, 100 units of output. To keep the analysis very simple, let us assume that both the wage level and price level are fixed. Such a case is analyzed in Figure 2.

Figure 2

The Effect of a Shift of 100 in Aggregate Demand

The answer to which one is naturally led by the current AS/AD analysis is the following: the shift aggregate demand of 100, price level assumed fixed, leads to a decrease in output of 100 and a new temporary equilibrium real output at Q^D .

This fixed price level case is the same case that is analyzed in the Keynesian Cross and IS/LM models. Implicitly, according to the standard textbook exposition of the AS curve, the adjustment shown in Figure 2 is assumed consistent with these models. It is not; the adjustment in Figure 2 is inconsistent with the derivation of the standard aggregate demand curve which already has included a dynamics supply demand interaction in the derivation of the AD curve. This can be most easily seen by comparing the Q^D equilibrium with the equilibrium that would be arrived at through use of a Keynesian Cross model which used to be the first introduction to

Mr. Keynes and the Classics Again

Keynesian economics that students had.³ In the Keynesian Cross model any exogenous shift in demand will be multiplied by some amount, the size of which is determined by the multiplier. So in the Keynesian Cross model, output will decrease by more than the exogenous shift in demand. The IS/LM model incorporates this multiplier effect with an interaction with the money market, but it still incorporates the multiplier interactive adjustment process.

This inconsistency is prima facie evidence that something has gone wrong in the simplification process which ultimately led to the current AS/AD interpretation of Keynesian economics and suggests that something much more fundamental is wrong with the current interpretation. To see what is wrong, it is helpful to consider alternative definitions of the AS and AD curves and the underlying micro foundations of the AS and AD curves.

An Alternative AD Curve

The traditional AD curve is derived from the IS/LM model and includes both the initial effect on output of a shift in the price level and the multiplier effect. From a microeconomics foundation point of view, this standard AD curve is a weird demand curve. It doesn't tell how much would be demanded at different prices, supply remaining constant. In the multiplier process, it includes an interaction with supply which brings the economy to supply demand equilibrium. The alternative AD curve I propose includes only the initial effect of a change in price on the quantity demanded. It tells us the quantity that will be demanded if the price level changes, aggregate supply remaining constant. Thus its elasticity is determined by the Keynes effect, the Pigou effect, and the international effect, and not by the size of the multiplier. The multiplier involves an interaction of aggregate supply and demand and thus is part of the disequilibrium adjustment process. My contention is that the multiplier process should *not* be part of the analysis of the shape of my AD curve. It follows that my alternative AD curve is always more inelastic than is the traditional AD curve.

A second difference between my alternative aggregate demand curve and the standard aggregate demand curve is that in my AD curve, aggregate demand is a function of aggregate supply: as aggregate supply changes, so, too, does aggregate demand, although, following Keynes, by a smaller amount than the shift in aggregate supply. Thus, shifts in aggregate supply cause shifts in my AD curve; only changes in the price level, supply remaining constant, cause movements along my AD curve. What I have done with my alternative definition is to separate out the multiplier effect, which is part of the disequilibrium adjustment process, from the microeconomic determinants of the AD curve.

My specification of the AD curve as including an interactive term with supply is clearly quite different than the standard analysis. It will also lead one to ask: How are these shifts to occur, since the traditional AS curve doesn't shift around? It is correct that the traditional AS curve doesn't shift around, but that is precisely what is wrong about the interpretation of the of the standard aggregate supply curve. Thus, let us now turn to a reconsideration of the AS curve.

An Alternative AS Curve

The traditional AS curve is assumed to be the aggregate summation of the individual firms' production function. Assuming capital fixed, aggregate supply is assumed to depend only on the amount of labor supplied and demanded which, in turn, depends on the real wage, the nominal wage relative to the price level. Thus in the traditional AS analysis, the Keynesian AS

³ This inconsistency has led some introductory texts to abandon the multiplier model and present only the AS/AD model. The inconsistency with the IS/LM model is more subtle, but since the IS curve is explicitly a goods market equilibrium curve, it cannot be used to discuss the disequilibrium dynamics of the goods market. It can only be used to discuss the disequilibrium dynamics of the interaction of the goods and the money market.

Mr. Keynes and the Classics Again

curve is upward-sloping due to a fixed nominal wage, and the Classical AS curve is perfectly inelastic at full employment output. (Various other shaped AS curves can be derived with alternative assumptions about the production function, relative flexibility of wages and prices, and elasticities of labor supplies.) If one assumes nominal wages are perfectly flexible, the Keynesian aggregate supply curve becomes identical to the Classical aggregate supply curve.

But Keynes explicitly stated that his difference with the classicals did not depend on the flexibility of nominal wages. Thus, the standard Classical model of the AS curve misses an important part of Keynes's thought. That missing part is to be found in the standard aggregate demand curve, which is derived from a model which incorporates an assumed dynamic interaction between aggregate supply and aggregate demand. The standard model hopelessly intertwines these relationships and in the process fails to point out the key elements of difference between classical and Keynesian thought. My proposed alternative demand curve separates out any assumed Keynesian adjustment process from its derivation, If it is to be added it must be added explicitly to the analysis of supply.

To do so one would make the following adjustment. A Keynesian would argue that the standard consideration of the AS curve misses an important aggregation problem. It implicitly assumes firms can fully coordinate their output decisions and produce maximum output unless there are fixed nominal wages, in which case that nominal wage institutional constraint is the only coordination problem. That assumption assumes away other types of coordination problems of the aggregate economy and macroeconomics; it effectively eliminates the Keynesian story of the multiplier process from the analysis.

The foundations of Keynesian economics requires a recognition that aggregate economies have serious problems coordinating their output (supply) decisions and often produce less than is technically possible because of concern that the demand for the product will not be there. This means that there must be a coordin Having separated out the analysis of effective demand from the derivation of the AD curve, one must explicitly include the possibility of demand influencing supply decisions as occurs in the analysis of effective demand in the analysis of supply. The easiest way to do this is to include another factor of production in the production function, calling that factor the degree of coordination.

$$Q_s = F(K, L, \text{Coordination})$$

According to the Keynesian thinking if the degree of coordination changes, output will change independently of the labor and capital inputs. Thus, in my interpretation of the Keynesian aggregate supply function there is an unstable element in the production function which causes the aggregate supply curve to shift. The Keynesian model must specify the nature of that unstable element, and what degree of interdependence there is between aggregate supply and demand. In the standard Keynesian model which underlies the derivation of the traditional AD curve, the coordination function is specified to be a function of expected demand which in turn is a function of expected supply which in turn is a function of expectation demand, and so on. The multiplier analysis provides a limit analysis of that coordination failure and determines where the equilibrium would be after the coordination failure has moved to a new equilibrium. This may or may not be the right analysis of coordination failure, but if one is to correctly capture the standard Keynesian analysis of effective demand in a meaningful AS/AD model one must incorporate this assumed coordination failure in the production function and hence in the AS curve.

Thus, the standard Keynesian analysis of aggregate supply is that it is a function of expected aggregate demand which, in equilibrium, for expositional ease, can be considered equal to actual aggregate demand at the firms' current supply decisions. This is the simplest Keynesian formulation, the formulation which underlies the Keynesian Cross. In it aggregate supply depends upon aggregate demand and aggregate demand depends upon aggregate supply. If, for

Mr. Keynes and the Classics Again

some reason, there is a shock in expected demand, supply will shift in response. But as supply shifts, expected and actual demand will also shift and the firms in the economy will find themselves in a coordination muddle. Eventually a new equilibrium will be arrived at where aggregate supply equals aggregate demand, but this equilibrium can be at a variety of levels of output.

This means that in the Keynesian analysis *there is not a one-to-one relationship between the number of workers used in the production process and the output of those workers*. Output depends upon the firms' expectation of aggregate demand, as well as the number of workers hired. Workers' marginal and average productivity can shift dramatically as firms' expectations of demand change.⁴ Thus in my construction the equilibrium arrived at is a type of boot-strap equilibrium determined by expectations.

This dependency of aggregate supply on aggregate demand does not affect the elasticity of the aggregate supply curve. As in the standard analysis, in my construction the elasticity of the AS curve depends upon the fixity of nominal wages and the technical nature of the production function. But in my alternative construction of the AS curve, the price elasticity of supply is only part of the story that needs to be captured and only one of the differences between Keynesian and classical analysis. The more important difference is the interrelationship between aggregate supply and demand. Keynesians assume that such an interdependence exists; classicals assume that it does not exist.

The difference between the two approaches can be seen by assuming both nominal wages and prices are fixed. It was this case that I considered in Figure 2. In the standard interpretation this case is analyzed as if there were a perfectly elastic supply curve with no specification of what causes the supply curve to be perfectly elastic; in my interpretation this case is analyzed as a disequilibrium case in which there is a shifting aggregate supply which reflects an assumed interrelationship between aggregate supply and demand. That assumed interrelationship is, or should be, the central difference between Keynesian economics and classical economics. It is a difference which is completely missing from the current AS/AD interpretation. My alternative construction allows its integration into the model.

KEYNESIAN AND CLASSICAL VIEWS OF THE AGGREGATE ADJUSTMENT PROCESS

Having defined the AS and AD curves in a way that one can use them to discuss aggregate disequilibrium adjustment, let me now specifically consider the adjustment process. I begin with the simplest case, the same one presented in Figure 2, in which the price level is fixed at P_1 and aggregate demand shifts by 100. I show this case in Figure 3. Notice that now, since the multiplier effect has been removed from the determination of the slope of the AD curve, it is appropriate to shift the AD curve by 100. But that is not the end to the adjustment process. We are in a disequilibrium with quantity supplied exceeding quantity demanded by 100.

⁴In my formulation, worker's productivity could fall in a recession whereas in the standard analysis one would expect worker's productivity to rise.

Figure 3

Fixed Price Level Adjustment in My Alternative Model

A key to understanding my interpretation is to recognize that **saying that the price level is fixed is not the same as saying that the supply curve is perfectly elastic.** It is, instead, saying that there are institutional constraints on the price level which prevent the economy from moving to equilibrium, P_e , Q_e . One cannot simply assume that the equilibrium output falls to Q_D without explaining what disequilibrium adjustment forces will push you there and keep you there. A fundamental question which both Classical and Keynesian economics must address is what happens to the economy when there exist institutional constraints which prevent, or simply slow, the movement to equilibrium via adjustments in the price level.

Let us discuss in detail some possible disequilibrium dynamics underlying the adjustment process. Specifically, given the assumptions of this case, what happens if the price level is fixed in disequilibrium at price P_1 so that the aggregate economy cannot move to the supply/demand equilibrium? This question is a question involving disequilibrium dynamics; it has little to do with comparative static analysis; it is the question that is posed and answered in the simplest presentation of the Keynesian model--the Keynesian Cross model. One of the reasons why, I believe, this initial exposition of Keynesian economics avoided the use of an AS/AD analysis and instead focused on the Keynesian Cross was that Keynes and the early Keynesians recognized that the question: How would income in the aggregate economy adjust to bring about aggregate equilibrium? would involve interdependent shifting of the curves as well as movement along curves. They chose in the Keynesian Cross model to focus only on the interdependent shifting.

Disequilibrium adjustment was not a topic that Classical economists focused on and the implicit Classical answer to the question of how the aggregate economy will adjust to equilibrium if the price level is institutionally constrained at a specific price was, and still is, that it will not adjust. The economy will stay in continual disequilibrium with the quantity supplied, Q_S , greater than the quantity demanded, Q_D , until the institutional constraint is removed.⁵ If Classical economists had posed this question they might have suggested that since the supply/demand analysis is a flow analysis, the stock of unsold goods would be continually

⁵ After reading an earlier version of this paper, Nancy Wulwick directed me to earlier literature on the AD curve. One of those papers was by A Rubin and D Birch (1982), who made essentially this point. Their paper is further discussed in David Smyth (1989), to which they responded (1989).

Mr. Keynes and the Classics Again

growing, and perhaps that growing stock would eventually break down the institutional constraints preventing the price level from falling. So the Classical answer might be that the stock disequilibrium would continually grow from the continual flow disequilibria, and, eventually, the excess of goods would break down the institutional constraints holding prices up.

Keynes posited a different disequilibrium adjustment response. Instead of assuming that unsold goods pile up, he argued that in response to disequilibrium, suppliers will decrease supply, shifting the supply curve back to S_1 , eliminating the excess supply. To show this decrease in supply one needs a shift factor in the production function. Keynes did not, but one might, argue that after the supply curve shifts, a new equilibrium will be reached at price P_1 and output q_1 . This, I believe, is the implicit adjustment process that most economists have in the back of their minds when they discuss this fixed price case as if it is a case of an infinitely elastic supply curve. After the supply shift there is no continual disequilibrium. Supply simply adjusts to meet demand.

The above disequilibrium adjustment is not a description of Keynesian analysis. In Keynesian analysis, in the aggregate, supply and demand are interrelated. As supply (output) is decreased, income falls and aggregate demand which is dependent on income falls further. That is why the multiplier process was central to Keynes' discussion of equilibrium. This interconnection means that as the aggregate supply curve shifts, so too does the aggregate demand curve. This interdependence of supply and demand is a type of coordination failure. P_1 , Q_1 is a potential equilibrium, but Keynes argued that the economy would not reach that equilibrium since as suppliers cut back their production, demand would decrease further.

To see the Keynesian adjustment process at work, let's follow the shifts through a number of interactions. We begin at a disequilibrium with the price level, P_1 , the quantity supplied, Q_S , and the quantity demanded, Q_D . In the Keynesian view this initial gap causes the aggregate supply curve to decrease from AS_0 to AS_1 . As the supply curve shifts, the aggregate demand curve would also shift, starting a multiplier process. The key Keynesian assumption in this adjustment process is that the demand curve shifts less than does the supply since the mpc is less than 1, so this dual shift has decreased the disequilibrium gap. As can be seen in Figure 3, it shifts back by the quantity b ($Q_S - Q_D$). Thus the disequilibrium gap has been reduced, but not eliminated. This means that there are further adjustments to AS_2 and AD_2 and so on. The two curves will keep shifting until, in the limit, supply and demand once again intersect. For example, if the mpc is .5, the multiplier is 2 and the final equilibrium is at output level $Q_0 = Q_S - 2(Q_S - Q_D)$. The final equilibrium output would depend on that multiplier process which is dependent on the nature of the interaction between supply and demand.

I should point out that the final Keynesian equilibrium arrived at from my analysis is a point on the traditional aggregate demand curve. Thus, in this case, my analysis arrives at the same equilibrium as does the traditional analysis. The only difference is that it makes the disequilibrium adjustment process which underlies that equilibrium clear. It only arrives at the standard equilibrium because it assumes a shifting supply curve.

It is central to one's understanding of the Keynes/Classics debate to understand that the standard analysis of the fixed price equilibrium (Q_D , P_1) is a fundamentally different equilibrium than the Keynesian fixed price equilibrium (Q_0 , P_1). Moreover, **it is important to recognize that Classical have no equilibrium when prices are fixed**. In the Classical case the supply curve does not shift at all even if there are fixed prices. This means that excess aggregate supply exists which will place a downward pressure on the price level and eventually lead to an equilibrium at Q_E , P_E . In the standard analysis of fixed price equilibrium and in the

Mr. Keynes and the Classics Again

correct Keynesian analysis of fixed price equilibrium, the excess supply will have been eliminated by the shifting aggregate supply curve so there will be no downward pressure on the price level at equilibrium. In both these cases excess aggregate supply is eliminated. At equilibrium (Q_0, P_1) and at equilibrium (Q_D, P_1) there is no downward pressure on the price level coming from excess aggregate supply, although there might be downward pressure coming from other forces. The traditional AS/AD analysis does not make this clear: The traditional AD curve is a fundamentally Keynesian construction which has already incorporated a Keynesian income adjustment process and a shifting supply curve. . It is logically inconsistent to use that Keynesian construction to talk about Classical price level adjustments.

Notice the fundamental difference in the Keynesian interpretation and the Classical interpretation. In the Classical interpretation a disequilibrium of $Q_S - Q_D$ will not cause any shift in either the AS or AD curve. Disequilibrium forces will build up to push the economy toward the equilibrium (Q_e, P_e) . But **in the Keynesian analysis that pressure toward equilibrium from aggregate supply exceeding aggregate demand is itself dissipated by an alternative adjustment mechanism.** The Keynesian equilibrium (in the case of fixed prices) has nothing to do with the slopes of the AS or AD curves; it has only to do with their interdependence.

In modern terminology, the argument that is being made here is that the Keynesian aggregate equilibrium is path dependent, by which I mean that the final equilibrium is dependent on the initial disequilibrium. **In the Keynesian model equilibrium cannot be analyzed separately from disequilibrium.** The Classical equilibrium is a special case of the Keynesian case which occurs when there is no initial disequilibrium. That is why (or should have been why) Keynes said that the Classical case is true, "assuming full employment." This path dependency of the aggregate equilibrium is what Keynes either meant, or should have meant, when he argued that his was the more general theory. When there are fixed prices, or, as I will show below, in almost all cases when there are less than instantaneously adjusting prices, **the short-run aggregate equilibrium is dependent on the initial disequilibrium.**

The Keynesian model which captures this path-dependent short-run equilibrium does not, or at least should not, deny the fact that there might be other dynamic forces operating, pushing the economy toward some long-run equilibrium. But, in what I call the Keynesian disequilibrium adjustment explanation, the normal forces that the Classical model focuses on are dissipated by the alternative income adjustment process. Because those forces are dissipated, Keynes' theory is the general short-run theory and the Classical theory is a special case of that short-run theory--the case in which no disequilibrium analysis is needed.

In the fixed price case, the key to determining the final Keynesian equilibrium at which the economy will arrive, and indeed to determining whether an equilibrium will exist, concerns the relative degree of interdependence of AS and AD. Since it concerns the interdependence of aggregate supply and aggregate demand, the comparative static analysis which assumes no interdependence cannot be used to solve for equilibrium income. That's why Keynesian economics switched models and why the simple Keynesian multiplier model does not correspond to the AS/AD model in which the AS curve is perfectly elastic. Such a model misses the entire point of the Keynesian adjustment process.

In the Keynesian model the marginal propensity to consume, modified by the interaction with the money market, is the dynamic interrelationship between the AS and AD curves. For example, if the mpc is .8, the aggregate demand curve shifts by .8 whenever the aggregate supply curve shifts by 1. The geometric relationship between the disequilibrium adjustment interpretation of the AS/AD model and the simple Keynesian AP/AE (Aggregate Production/Aggregate Expenditure) model is demonstrated in Figure 4.

Mr. Keynes and the Classics Again

Figure 4

In it you can see how the disequilibrium adjustment story that is used in the Keynesian Cross model to explain the effect of a downward shift in aggregate expenditures corresponds to my AS/AD disequilibrium story. In both, the initial disequilibrium is caused by a downward shift in demand which causes excess supply. In both, that excess supply causes a reduction in supply which, in turn, causes a further reduction in demand. In both, this process continues until supply and demand are once again in equilibrium. In both, the multiplier process determines the final equilibrium. **Since a movement along the Aggregate Production curve (the 45° line) corresponds to a shift of the AS curve and a movement along the Aggregate Expenditure (AE) curve corresponds to a shift of the AD curve**, in order to provide the same disequilibrium adjustment story as does the Keynesian Cross model, the AS/AD model must include shifting curves.

Notice that in my analysis the initial disequilibrium position determines the final equilibrium that is reached, and many equilibria are possible, depending on what the initial disequilibrium is. This was one of Keynes' messages, and that message has been lost in the

Mr. Keynes and the Classics Again

current AS/AD interpretation of Keynesian economics. My alternative model captures Keynes' message.

FLEXIBLE WAGES AND PRICES AND THE KEYNESIAN MODEL

The discussion above is simply a method of showing that the current, in-vogue interpretation of AS/AD is inconsistent with the Keynesian multiplier model. A typical reaction has been: "So what? The AS/AD model allows flexible wages and prices and thus naturally arrives at a quite different interpretation of Keynesian economics and macroequilibrium." I contend that this reaction misses the central point of the above analysis; **unless the AS/AD model includes a shifting AS curve and an interdependent AS/AD analysis, the AS/AD model totally guts the Keynesian insight into the economy**, even when the price level is flexible.

To see that this is the case, let us assume for the moment that the price level is partially, but not instantaneously, flexible. For simplicity, let us also assume that the wage level is perfectly flexible. This assumption allows us to focus on the role that price level flexibility plays in the Keynesian model. (Later I will relax this assumption.)

This perfect wage flexibility assumption makes the relevant aggregate supply curve perfectly inelastic since the real wage will never deviate from equilibrium when the price level changes. No unemployment can result from the real wage (W/P) being too high since, by assumption, the real wage adjusts to maintain equilibrium in the labor market. **But that doesn't mean that the equilibrium income (the real goods and services available) cannot fall substantially.** The miscoordination in the goods market reduces the productivity of both capital and labor. The mere fact that there is no unemployment does not mean that people's real income can't fall. Since my Keynesian production function includes a coordination factor,

$$Q = F(K, L, \text{Coordination})$$

a decrease in coordination, which is assumed to occur whenever an initial disequilibrium occurs and the aggregate price level is less than perfectly flexible, is directly associated with a decrease in both the profit rate and the wage rate. In my construction, the flexibility of the nominal wage is a red herring. It only determines how the decrease in real income resulting from miscoordination will be spread between the workers and owners of capital. A fixed nominal wage simply means that instead of real wage income falling by 10% when each worker gets a 10% cut in wages relative to the price level, 10% + E% of the workers will get laid off, the additional E% being the real wage effect. In the short run, I conjecture that in the short run the real wage effect is relatively small; the action in my interpretation of Keynes is in the product market.

Given this assumption of instantaneous wage flexibility, the short-run equilibrium will be determined by a combination of two disequilibrium adjustment processes: the Keynesian income adjustment process and the Classical price adjustment process. The nature of that combination is shown in Figure 5:

Mr. Keynes and the Classics Again

Figure 5

The Keynesian Model with Partially Flexible Wages and Prices

Once again, assume we start out at disequilibrium gap of $Q_S - Q_D$ at price level P_1 . If the price level is perfectly flexible, this disequilibrium gap will be eliminated instantaneously and there will be a new full employment equilibrium at (P_0, Q_S) . The case of perfectly flexible prices is consistent with the Classical interpretation because perfectly flexible prices rule out any Keynesian multiplier process.

But what happens if the price level is flexible, but less than perfectly flexible? In that case, there are two disequilibrium adjustment processes occurring simultaneously: the Keynesian income adjustment process which causes the interdependent shifting of the AS curve and AD curve (represented by arrow A), and the Classical price adjustment process (represented by arrow B) which causes a downward movement along the AS curve and along the AD curve. These two adjustment processes will be occurring simultaneously. **A correct Keynesian/Classical synthesis involves integrating the two adjustment processes together.**

To determine what the equilibrium will be in this synthesis case, we must make an assumption about the relative speeds of adjustment and the slope of the aggregate demand curve. In the real world these relative speeds of adjustment are likely to be functions of other variables, but for simplicity I shall assume the relative speeds of adjustment constant.

Specifically, let r = the speed of percentage income adjustment/speed of percentage price level adjustment;

$$r = \frac{dQ_s / Q_s / dt}{dP_s / P_s / dt} = \frac{dQ_s / Q_s}{dP_s / P_s} = \frac{A}{B} \frac{P_s}{Q_s}$$

Mr. Keynes and the Classics Again

Also let E_D = the absolute value of the elasticity of the aggregate demand curve.

$$E_D = \left| \frac{dQ_D/Q_D}{dP_D/P_D} \right| = \left| \frac{dQ_D}{dP_D} \frac{P_D}{Q_D} \right|$$

Say that initially the economy starts out at disequilibrium, $Q_S - Q_D$, or a percentage quantity gap of $(Q_S - Q_D)/Q_S$ and a percentage price gap of $(P_1 - P_0)/P_1$. Normalizing Q_S and P_1 to 1 to aid in geometric exposition, the two alternative adjustment processes which determine the ratio, r , are approximately represented by the ratio of the length of the vectors A and B. As the supply curve shifts back by A, the price level falls by B. These are, in principle, measurable quantities so the appropriate combination of the two models can be made on empirical grounds.

The elasticity of demand tells us the dampening effect that price level adjustment will have on the simple Keynesian multiplier process. Thus the fall in the price level increases the quantity demanded by the vector C. So the net shift back in demand will be $b(A - C)$. Thus, in Figure 5, as the supply curve shifts back along arrow A, the price level falls so the price/quantity adjustment is along arrow D. So instead of shifting to Q_D , the initial shift is decreased by $Q_1 - Q_D$, the length of which is related to the slope of the demand curve.

In the limiting case where the Keynes effect, Pigou effect, and international effect are inoperative, the aggregate demand curve is perfectly inelastic and the relative degree of disequilibrium price/quantity adjustment makes no difference. Thus this case reduces to the simple Keynesian model. It follows that the Keynesian model need not assume perfectly fixed prices to arrive at its strong conclusion, as Keynes pointed out in **The General Theory**; the model also reduces to the simple Keynesian model when the nominal sector and the real sector are independent of one another so that the price level has no effect on the quantity of aggregate real demand.

In the case of a **partially** flexible price level, the simple Keynesian model's results are modified but the central conclusion --that the short-run equilibrium is dependent on the disequilibrium adjustment path and cannot be derived by a comparative static model which cannot account for path dependencies--is not. Thus, **the traditional comparative static AS/AD model--which assumes equilibrium is determined independently of the initial disequilibrium--cannot be used in any case other than the infinitely fast price level adjustment--a case that has no relevance to reality.**

But that doesn't mean that the introduction of a flexible price level model doesn't change the Keynesian analysis. As the price level falls, the amount by which the AD curve shifts back decreases, but it nonetheless shifts back. The final equilibrium is at a point like P_n, Q_n which involves a combination of the price adjustment process and the quantity adjustment process.

To see the actual net adjustment in this synthesis case, we need to take the limit of the two adjustment processes occurring simultaneously. Assuming the initial percentage quantity disequilibrium is $(Q_S - Q_D)/Q_S$, the first step in the demand adjustment will be to Q_2 which equals

$$b(1 + E_D/r)(Q_S - Q_D)$$

where:

Mr. Keynes and the Classics Again

$$E_D/r = \frac{\frac{dQ_D}{Q} / dP_D/P}{\frac{dQ_S}{Q} / dP_S/P} = -\frac{dQ_D}{dQ_S}$$

which is the quantity demand adjustment due to the fall in the price level. The next step in that amount squared and so on.

(I'm still working on this formula; the general form is right, but I've got to work on the exact expression of it.)

Taking the limit of this process we can see that the net change in real income when both adjustment processes are operative will be:

$$\lim_{n \rightarrow \infty} b \left(1 + \frac{E_D}{r}\right) + \left[b \left(1 + \frac{E_D}{r}\right)\right]^2 + \left[b + \frac{bE_D}{r}\right]^3 + \dots + \left[b + \frac{bE_D}{r}\right]^n = \left[\frac{1}{1 - b + bE_D/r} \right]$$

This solution nicely accords with intuition. For example, let's say that the price adjustment process is instantaneous, so that $r = 0$. In this case the net multiplier approaches zero. Real income will not change; all adjustment will be accomplished via price level adjustment. This is one route to the Classical model.

Alternatively when the elasticity of aggregate demand approaches ∞ , so that it becomes perfectly elastic, the net multiplier approaches zero. This is a case in which the Keynes effect, the Pigou effect, and the international effect are extremely effective, and is an alternative route to the Classical model.

On the other extreme, when price level is institutionally fixed, so that $r = \text{infinity}$ or when $E_D = 0$ the net multiplier reduces to the simple Keynesian model.

These are, of course, the extremes. In the most likely case, when both the price adjustment and quantity adjustment processes are operative, the synthetic solution will be somewhere between the two extremes. For example, say that:

$$b = .8$$

$$r = 5$$

$$E_D = .5$$

In the absence of any price adjustment the multiplier will be 5. With this partial price level adjustment the net multiplier is reduced to 3.6. As expected, the addition of price flexibility reduces the net shift in real income. The smaller either r or E_D is, the more the multiplier is reduced. Thus, the reduced form model:

$$\Delta Y = \left(\frac{1}{1 - b + bE_D/r} \right) \Delta A$$

where ΔA = change in
autonomous expenditure

Mr. Keynes and the Classics Again

I argue, is the true synthesis model which captures both the Keynesian insight about quantity adjustment and the Classical insight about price adjustment.

The vector, P_1 , which traces out the disequilibrium path of prices associated with the fall in income, has the same shape as the traditional aggregate supply curves, but within my framework the interpretation is fundamentally different than it is in the traditional model. This line traces shifts of the aggregate supply curve, not movements along an aggregate supply curve. Its shape is governed by the relative speeds of adjustment of income and price, not by the fixity of the nominal wage, nor by the elasticities of the supply or demand for labor. The action in my Keynesian interpretation is in the product market, not in the labor market.

DIFFERENTIAL WAGE PRICE FLEXIBILITY

To say that the action in my interpretation of Keynes is in the product market, not the labor market, does not mean that a sub-scene of the major action does not involve the labor market. It is only to say that to focus on it is the equivalent of focusing on tingles in the foot when describing sex. However, since the economics profession has maintained a foot fetish for quite a while, it is probably worthwhile to consider how labor market considerations affect the result. To do so, consider Figure 6.

Figure 6

Adjustment with Differential Wage and Price Flexibility

When nominal wages are less flexible than nominal prices, real wages can deviate from the equilibrium real wage. In this case as prices fall, real wages rise which causes the AS curve to be upward sloping following the normal Keynesian assumptions about labor supply. The result is the normal AS/AD model that was presented in Figure 1, and hence Figure 6 presents the same case as Figure 1. In it the economy experiences a demand shock, shifting the AD curve from AD_0 to AD_1 at price level P_1 . This begins the Keynesian disequilibrium adjustment process-- shown by interactive shifting of the AS and AD curves and a price adjustment process which this time causes movements along the AD and AS curves. Both these movements decrease the amount of income adjustment that is needed to bring the economy into equilibrium.

Thus, fixed, or slowly adjusting, nominal wages, while causing some direct unemployment due to a too-high real wage, reduce the needed decrease in real income necessary to bring about equilibrium. As they do so, fixed nominal wages can help stabilize the real income in the economy.

Mr. Keynes and the Classics Again

Analyzing exactly how fixed nominal wages might stabilize the economy is complicated. Differential flexibility of wages and prices does not necessarily affect the analysis in the same way as does the elasticity of demand, because it is not clear whether a movement along the supply curve affects the interrelationship between supply and demand any differently than does a shift of the aggregate supply curve. A decrease in real income caused by a movement along the AS curve and an equal decrease in real income caused by a shift of the AS curve both involve the same decrease in real income and hence will likely affect the shift in aggregate demand similarly. Changes in the relative wage/price ratios have no direct real effects through the goods market.

But this is not necessarily the case if there are distributional effects as there are in the Kalecki interpretation of the Keynesian model. In this case, if wages adjust more slowly than prices do, a fall in the price level will increase the real wage which will tend to hold up aggregate real demand. In the limiting case where profit income is not spent, and 100% of wage income is spent, the net multiplier becomes:

$$\Delta Y = \left(\frac{1}{1 - b + bE_D/r + bE_S/r} \right) \Delta A$$

where S is some outside
real shock

Notice that in this case, from an aggregate standpoint, slowly adjusting wages reduces the size of the net multiplier and hence reduces the decrease in income that would otherwise take place, just as Keynes said it would.

But that is not the only effect a slowly adjusting wage level is likely to have. Since the wage level is an important determinant of the price level, slowly adjusting wages will likely mean more slowly adjusting prices which will decrease r and increase the size of the net multiplier. Hence the net effect is unclear.

SOME OBSERVATIONS ABOUT THE MODEL

I will conclude with a few observations about my alternative Keynesian model. The first is that the multiplier can be less than one so that the final shift in real income can be less than the initial shift. For example, say price adjustment is relatively fast so that r is small and the elasticity of demand is high. In that case, the final equilibrium shift in real income can be less than the initial gap.

Second, I should point out that the model says nothing about optimality of the equilibrium. It only says that the short-run equilibrium to which the economy gravitates is dependent on the initial disequilibrium and the disequilibrium adjustment process. Because of this path dependency, the AS and AD curves can intersect even at a level of output which involves an underutilization of resources compared to what there would have been had there been perfect coordination. Whether this underutilization of resources involves unemployment of labor or capital depends on issues that are not addressed in this paper.

A third point that should be made is the tremendous ambiguity of the concept of "the real wage." Normally it is taken to mean the wage/price ratio in a model with only capital and labor and no coordination problem, so that it is the equivalent of the wage/profit ratio. In such a model, to say the real wage is too high is the equivalent of saying that the profit rate was too low. Once one adds the coordination element to production, the term "real wage" is no longer a sufficiently descriptive term. In such a model both the real wage and the real profit rate could be too high so that the "real supply price" is too high. Similar definitional problems occur if one adds an international sector or third input to the model. The fixity of the nominal wage and

Mr. Keynes and the Classics Again

hence the flexibility of the relative wage does have a role to play in determining whether there will be extremely low relative wages or whether there will be unemployment. But, in my interpretation, the existence of unemployment is not necessary to the existence of a Keynesian coordination problem.

A fourth point I should mention concerns the stability of the Keynesian short-run equilibrium. Although there is no aggregate disequilibrium in the goods market at the Keynesian equilibrium, and hence no downward pressure on the price level associated with that equilibrium, there may be other forces which are pushing down on the wage and price level. For example, if as opposed to all wages being equally flexible, some were more flexible than others, there may be downward pressure on nominal wages because of internal competition in the labor market which this aggregate model cannot capture. Thus wages and prices could be falling even if there is no excess aggregate supply of goods or labor. Whenever there is unemployment of any resource, one would expect there to be some downward pressure on the nominal price of that resource. But that downward pressure need not be the result of any aggregate disequilibrium. Thus, in the long run there may be forces which push the economy to a long-run non-path-dependent equilibrium, but these forces are not captured in an aggregate model.

Fifth, the model is a short-period model, by which is meant a model long enough for aggregate supply and demand to come to an equilibrium, given that the expectations have a chance to play themselves out. It is an equilibrium marked by an expectational conundrum in which suppliers would like to produce more if they felt the demand were there and workers would like to work more, but the demand is not there. At this equilibrium although there is no downward pressure on the price level, there may be a pressure to get together and provide each other assurances of demand and to move to a preferable equilibrium.

Sixth, the model is not a model of equilibrium income; it is a model of fluctuations in income, given an initial disequilibrium. This, I contend, was, or should have been, Keynes's central message--that it is inappropriate to assume the optimal income will be arrived at in the short run, given the disequilibrium. Keynes argued that the level of income is determined by expectations and deviations of expectations from predictions. In Keynes' view, and in the model, in the aggregate, expectations of demand are partially self-fulfilling. The shifts in demand and supply and the repercussions occur too often and too fast to make the long run properties of the economy of significant relevance, other than as something to keep in the back of one's mind.

Seventh, it is entirely possible that there are dynamic interactions in the economy that this specification of the model does not capture. For example, the falling price level could wreak havoc with the financial markets and increase the degree of miscoordination in the economy. In this case aggregate supply would itself be a declining function of the change in the price level.

Eighth, the policy prescriptions that flow from this model are Keynesian-like, but they deviate from most Keynesian models in an important way. The policy problem in this model is not that fluctuations in income occur. Such fluctuations may or may not be desired fluctuations. That issue isn't addressed. The policy problem that this model captures is that because of the interdependent aggregate supply and demand, any fluctuation in demand changes the long-run equilibrium, and price level adjustment will not bring the economy back to the original equilibrium.

The final point I should make concerns the exposition of the model and the ideas. I fully recognize that talking about simple geometric models is not the way that most serious debates about underlying theory in economics generally are presented. But, I believe, it is precisely the loss of the visual intuition that has led macroeconomics to its current state. Ultimately our understanding of the macroeconomy is based on stories. It is time we started dealing with stories we can understand rather than losing ourselves in technique-oriented stories which transcend our understanding.

Mr. Keynes and the Classics Again

There are, of course, an almost infinite number of ways in which this model can be extended. But most of these additions, I believe, will not change the interpretative vision underlying this model. I ask the reader not to let the simplicity of the above model undermine its significance. If one accepts its basic premises, it means that what just about every introductory and intermediate textbook teaches about aggregate equilibrium is wrong, as are many of the higher level stories which macro theorists tell.

Mr. Keynes and the Classics Again

APPENDIX

The change that I am suggesting in the Keynesian model is that supply be specifically considered. The Keynesian model, in my interpretation, is not an aggregate demand model; it is a model of both aggregate supply and aggregate demand. Let me show the algebra for the simplest model.

In the traditional interpretation, the model is written as follows:

$$C = a + bY$$

$$I = I_0$$

$$Y = C + S$$

$$S = I$$

and solved

$$Y = I/(1 - b) (a + I_0)$$

Nowhere in the model specification is it made clear what is aggregate supply or what is aggregate demand. Somehow the two merge into one.

In my specification, the distinction must be made. Thus the simplest model would be specified on both the demand side and the supply side: specifically

$$C_D = a + bY_S$$

$$I_D = I_0$$

$$Y_D = C_D + I_D$$

$$I_S = I_D^{E_D}$$

$$C_S = C_D^{E_D}$$

$$Y_S = I_S + C_S$$

C_D = demand for consumption goods

Y_S = real output supplied

I_D = demand for investment goods

Y_D = real output demanded

I_S - supply of investment goods

$I_D^{E_D}$ = expected demand for
investment goods

C_S = supply of consumption goods

$C_D^{E_D}$ = expected demand for
consumption goods

Since in equilibrium:

$$I_D^{E_D} = I_D$$

and

$$C_S = C_D^{E_D}$$

and

$$Y_S = Y_D$$

the model reduces to the same result as does the standard model, specifically

$$Y = (1/1-b) (a + I_0)$$

The difference is in the interpretation. My specification makes clear the role that demand expectations of suppliers plays in the adjustment process and in determining the aggregate

Mr. Keynes and the Classics Again

equilibrium income. This becomes important when one adds a specific model of production to this simple case. In the traditional case:

$$Y/P = F(N)$$

$$W/P = F'(N)$$

$$N = J(W/P)$$

In my specification,

$$Y/P = F(N, Y^E_D/P)$$

where Y^E_D = expected aggregate real demand

$F'(N)$ is the marginal product of labor but it is a partial derivative dependent on the expected real demand. If expected real demand is high, real output will be high, which will significantly raise the demand for labor and increase the marginal productivity of labor; if expected real demand is low the demand for labor will fall significantly, and decrease the marginal productivity of labor. As long as the real wage is perfectly flexible, and the equilibrium wage doesn't become negative, there will always be full employment in the labor market, but that full employment could be at a very low level of income.

In my interpretation monetary and fiscal policy work through their effect on expected demand; thus they work through the production function, as well as through their effect on demand.

In my specification, monetary and fiscal policy play a direct role in production; they serve a coordinating function, achieving adjustments in the aggregate that individuals alone can't make.⁶ (Say that a sudden sense of fear hits producers. They can all believe this fear is irrational, but they believe other producers are acting on it, so they rationally act on it. The economy would fall into a self-fulfilling recession.

Expansionary government aggregate policy can directly affect the expectations of producers and thereby avoid the recession. Thus monetary and fiscal policy operate as much, or more, on the minds of people than they do on aggregate demand and the technical economy. Real-world practitioners of monetary and fiscal policy understand this; it is academic economists who have not understood it.

⁶I first made the argument back in Colander, (1979)

Mr. Keynes and the Classics Again

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**Mr. Keynes and the Classics Again:
An Alternative Interpretation***

David C. Colander

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Mr. Keynes and the Classics Again

$$r = \frac{dQ_s / Q_s / dt}{dP_s / P_s / dt} = \frac{dQ_s / Q_s}{dP_s / P_s} = \frac{A}{B} \frac{P_s}{Q_s}$$

$$E_D = \left| \frac{dQ_D / Q_D}{dP_D / P_D} \right| = \left| \frac{dQ_D}{dP_D} \frac{P_D}{Q_D} \right|$$

$$b \left(1 + \frac{E_D}{r} \right) (Q_s - Q_D)$$

$$= \frac{\frac{dQ_D}{Q_D} / dP_D / P}{\frac{dQ_s}{Q_s} / dP_s / P} = - \frac{dQ_D}{dQ_s} = -C$$

$$\frac{E_D}{r}$$

$$\lim_{n \rightarrow \infty} b \left(1 + \frac{E_D}{r} \right) + \left[b \left(1 + \frac{E_D}{r} \right) \right]^2 + \left[b + \frac{bE_D}{r} \right]^3 + \dots + \left[b + \frac{bE_D}{r} \right]^n = \left[\frac{1}{1 - b + bE_D/r} \right]$$

$$\frac{\infty}{E_D}$$

$$\Delta Y = \left(\frac{1}{1 - b + bE_D/r} \right) \Delta A$$

$$\Delta Y = \left(\frac{1}{1 - b + bE_D/r + bE_s/r} \right) \Delta A$$