Linear Algebra
Exam 2
Spring 2007

April 19, 2007

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators are not permitted.

1. **Define: basis**.

2. **Define: column space**.

3. Find an LU factorization of the following matrix.

$$A = \begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$$

4. In what context is an LU factorization useful?

5. Compute the determinant of the following matrix. The fewer steps you make in the computation the more points you will be awarded (but please do show your work).

$$G = \begin{bmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{bmatrix}$$

6. **True or False:** Justify each answer by citing an appropriate definition or theorem. If the statement is false and you can provide a counterexample to demonstrate this, then do so. If the statement is false and be can slightly modified so as to make it true then indicate how this may be done.

   - If one row of a square matrix A is multiplied by k to produce matrix B, then $\det B = \frac{1}{k} detA$.

   - $\mathbb{R}^3$ is a subspace of $\mathbb{R}^4$.

   - If B is an echelon form of a matrix A, then the pivot columns of B form a basis for $ColA$.

   - Since the coordinate mapping is one-to-one, if a set of vectors is linearly independent then their image under the coordinate mapping is also linearly independent.

   - If A is an $n \times n$ matrix and A is invertible then $NulA$ contains infinitely many vectors.

7. The following set of vectors, $S = \{t + t^2, 3, 6 + t, t^2\}$, spans $\mathbb{P}_2$. Using the elements of S give a basis for P_2. Use as few computations as necessary, justify your solution.

8. The following set of vectors is not a basis for $\mathbb{R}^3$. Show how this set can be expanded to form a basis for $\mathbb{R}^3$.

$$\mathbf{b_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\mathbf{b_2} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

9. Given two bases, $\mathcal{B}, \mathcal{C}$, for the same vector space V, the change of coordinates matrix,

$$\underset{\mathcal{C}\leftarrow\mathcal{B}}{P} = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$$

and $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$. Find $[\mathbf{x}]_{\mathcal{B}}$. (Be careful.)

10. Prove that the null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.

11. Suppose that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is a linearly dependent spanning set for a vector space V. Show that each $\mathbf{w}$ in V can be expressed in more than one way as a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$.