Calculus II - Exam 1 - Fall 2016

October 6, 2016

Name:
Honor Code Statement:

Directions: Complete all problems. Justify all answers/solutions. Calculators, texts or notes are not permitted. The value of each problem is indicated in brackets. Please remember the writing expectations that we've discussed in class while keeping the time constraint in mind.

1. [10 points] Upon retiring I will have shoved some money under my mattress. After that I will take from the money under my mattress so that the total present under the mattress decreases at the continuous rate of 10% per year. When will the mattress's stash fall to one-fifth of its initial value?

2. [5 points each] **Find** the derivative of each of the following functions. **Indicate** the method/rule used.

(a) $p(x) = \int_{x^2}^{5} sin(2t)dt$

(b) $y = \log_2(x \log_5 x)$

(c) $y = xe^{-x}$. After finding the first derivative, find the $1,000^{th}$ derivative.

3. [5 points each]

    (a) State the Fundamental Theorem of Calculus, Part 2.

    (b) Define the term *one-to-one*. Give an example of a one-to-one function and justify that the example is one-to-one.

4. [5 points each] Determine the following limits. Identify any indeterminate forms.

(a)
$$\lim_{s \to 0} \frac{4^s - 3^s}{s}$$

(b)

$$\lim_{n \to \infty} (\frac{n+p}{n})^n, \quad where \ p > 0$$

5. [5 points each] Evaluate the integral.

$$\int \frac{\log_7 x}{x}\,dx$$

6. [5 points] Prove that $\lim_{x\to\infty} \ln x = \infty$.

7. [5 points] Napier's Inequality states that for $b > a > 0$, we have $\frac{1}{b} < \frac{\ln b - \ln a}{b - a} < \frac{1}{a}$.
   Explain how the picture below establishes this result.