# Calculus II - Exam 2 - Techniques of Integration

October 17, 2013

**Name:**
**Honor Code Statement:**

**Additional Honor Statement:** I have not observed another violating the Honor Code._____

**Directions: Justify** all answers/solutions. Calculators are not permitted. You may use the table of trigonometric identities given on the last page. Each problem is worth 10 points, except for the last, which is worth 5 points. If you need extra space, use the blank white paper provided.

1. $$\int \frac{4x}{36 - x^2} dx$$

2.

$$\int \frac{\cos(x) + \sin(2x)}{\sin(x)} dx$$

3.

$$\int_0^{2\pi} t^2 \sin(2t) dt$$

4. What is the partial fraction decomposition of the following quotient? (Note that I'm not asking for the antiderivative.)

$$\frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)}$$

5. Determine if the following integral converges. If it converges, determine to what it converges.

$$\int_{-2}^{14} \frac{dx}{\sqrt{x+2}}$$

6. Determine if the following integral converges. If it converges, determine to what it converges.

$$\int_0^\infty \frac{x}{x^2+9}dx$$

7. Use the Direct Comparison Test to determine whether or not the following integral converges.

$$\int_{103}^{\infty} \frac{1}{x^2 + 5x - 109} dx$$

8. When we considered $p$-integrals, we restricted our attention to $p > 0$. Why did we restrict our attention in this manner?

# Trigonometric Identities

### Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

### Double-angle formulas

- $\sin(2x) = 2\sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

### Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

### Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$