Calculus II - Exam 2 - Techniques of Integration

October 20, 2016

Name:
Honor Code Statement:

Additional Honor Statement: I have not observed another violating the Honor Code._____

Directions: Justify all answers/solutions. For each of the first five problems, find the integral and **identify** the technique of integration used. Calculators are not permitted. You may use the table of trigonometric identities given on the last page. Each problem is worth 10 points. If you need extra space, use the blank white paper provided.

1. $$\int \sin(2\theta)\sin(6\theta)d\theta$$

2.

$$\int \frac{(\ln x)^2}{x^3} dx$$

3.

$$\int_0^1 \sqrt{x - x^2} dx$$

(Hint: begin by completing the square.)

4.

$$\int \frac{x}{(x+4)(2x-1)}dx$$

5.

$$\int_0^\infty \sin x \cdot e^{\cos x} dx$$

6. Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_0^{\pi} \frac{\sin^2 x}{\sqrt{x}} dx$$

7. Explain the following figure and the equation that goes with it.

$$\int_a^b \ln x \; dx = b \ln b - a \ln a - \int_{\ln a}^{\ln b} e^y \; dy = (x \ln x)|_a^b - (b - a) = (x \ln x - x)|_a^b$$

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2\sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$