Calculus II - Exam 2 - Spring 2012
Techniques of Integration

March 22, 2012

Name:
Honor Code Statement:

Directions: **Justify** all answers/solutions. Calculators are not permitted. You may use the table of trigonometric identities given on the last page. Each problem is worth 10 points. If you need extra space, use the blank white paper provided. **NOTE:** Please choose one of Question 4 and Question 5 to complete – indicate the one you omit by placing a "slash" through it.

1. Evaluate each of the following integrals. If there is a particular technique that you use, name it.

   (a) $\int x \sec^2(x)\; dx$

(b) $\int \tan^2 x \sec^4 x \; dx$

(c) $\int_0^1 \frac{1}{(x^2+1)^2}\ dx$ (Hint: use a trigonometric substitution.)

(d) $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\ dx$

2. Determine whether the following integral converges or diverges.

$$\int_e^\infty \frac{1}{x(\ln(x))^3}\,dx$$

3. Use the Direct Comparison Theorem to determine whether the following integral is convergent or divergent.

$$\int_1^\infty \frac{2+\sin(x)}{x^2}\,dx$$

4. Find the area of the region bounded by the given curves.

$$y = x^2 e^{-x}, \quad y = x e^{-x}$$

5. We can extend our definition of average value of a continuous function to an infinite interval by defining the average value of F on the interval $[a, \infty)$ to be

$$\lim_{t \to \infty} \frac{1}{t-a} \int_a^t f(x)\, dx.$$

Find the average value of $f(x) = \sin(x)$ on the interval $[0, \infty)$.

Trigonometric Identities

Addition and subtraction formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double-angle formulas

- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
- $\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$

Half-angle formulas

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

Others

- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$